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Solvable Groups -

A Group G is said to be solvable or soluble if there exits a chain of subgroups

$$\{e\} = H_0 \subseteq H_1 \subseteq H_2 \subseteq \cdots\cdots \subseteq H_n = G \quad -(1)$$

such that each H_i is a normal subgroup of H_{i+1} and H_{i+1}/H_i is abelian $\forall\ i = 0, 1, 2, \cdots n-1$

Also then the series (1) is refered to as solvable series of G.

Thus G is solvable if it has a normal series $(H_0, H_1, H_2 \cdots, H_n)$ such that its factor groups are abelian.

Theorem - 1

A subgroup of a solvable group is solvable.

Proof -

Let H be any subgroup of a solvable group G.

Since G is solvable,

$$G^{(n)} = \{e\} \text{ for some +ve integer } n.$$

Now, $H \subseteq G \Rightarrow H' \subseteq G' \Rightarrow (H')' \subseteq (G')'$

ie $H^{(2)} \subseteq G^{(2)}$

Continuing like this we get $H^{(n)} \subseteq G^{(n)} = \{e\}$

$\Rightarrow H^{(n)} = \{e\}$

$\Rightarrow H$ is solvable.

Proved